TITLE QUANTUM MECHANICAL STABILIZATION OF MINKOWSKI SIGNATURE WORMHOLES

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# Quantum mechanical stabilization of Minkowski signature wormholes.

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## Abstract

When one attempts to constuct classical wormholes in Minkowski signature Lorentzian spacetimes violations of both the weak energy hypothesis and averaged weak energy hypothesis are encountered. Since the weak energy hypothesis is experimentally known to be violated quantum mechanically, this suggests that a quantum mechanical analysis of Minkowski signature wormholes is in order. In this note I perform a minisuperspace analysis of a simple class of Minkowski signature wormholes. By solving the Wheeler-de Witt equation for pure Einstein gravity on this mini-superspace the quantum mechanical wave function of the wormhole is obtained in closed form. The wormhole is shown to be quantum mechanically stablized with an average radius of order the Planck length.

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## 1. Introduction.

The past year has seen a resurgence of interest in wormholes constructed on Minkowski signature Lorentzian spacetimes. This subject is quite distinct from the Euclidean wormholes that are being considered as possibly significant contributions to Euclidean quantum gravity. Minkowski signature wormholes have recently been considered by Morris and Thorne [1], by Morris, Thorne, and Yurtsever [2], and by the present author [3,4]. A major result of these investigations is that at the classical level violations of the weak energy hypothesis and the averaged weak energy hypothesis are guaranteed to occur at the throat of Minkowski signature wormhole.

On the other hand it is experimentally known that quantum mechanics violates the weak (and strong and dominant) energy hypotheses. The best known situation in which this happens is the Casimir effect [5]. Whether quantum mechanics permits violations of the averaged weak energy hypothesis is presently unknown. This suggests that quantum mechanics may be the key to interesting wormhole physics in Lorentzian spacetime, and it is this question that I shall investigate in this paper.

I shall present a quantum mechanical analysis of a minisuperspace that describes a class of Minkowski signature wormholes. The calculation proceeds by considering the path integral formulation of (pure) quantum gravity and brutally truncating the space of metrics. The Einstein Hilbert action is evaluated on this minisuperspace, and the corresponding Lagrangian and Hamiltonian are extracted. The resulting minisuperspace version of quantum gravity can be solved in closed form. The expectation value of the wormhole radius shall be calculated to be of order the Planck length. Since the classical Einstein field equations are irrelevant for purposes of the present calculation no attempt to satisfy them will be made.

The generalization to quantum mechanical wormholes based on surgically modified Schwarzschild and Reissner Nordstrom spacetimes is *in principle* straightforward, but is technically messy, and I am unable to obtain exact closed form wave functions for this important case. The introduction of matter is again *in principle* straightforward, but only approximate wavefunctions can be obtained.

## 2. The Einstein Hilbert action in mini superspace.

To construct the class of wormholes of interest, consider two copies of Minkowski space. Remove from each copy identical four dimensional regions $\Omega_1$ and $\Omega_2$. One is left with two geodesically incomplete manifolds with boundaries given by the hypersurfaces $\partial\Omega_1$ and $\partial\Omega_2$. Now identify these two hypersurfaces (*i.e.*, $\partial\Omega_1 \equiv \partial\Omega_2$). The resulting spacetime $\mathcal{M}$ is geodesically complete and posesses two asymptotically flat regions connected by a wormhole. The throat of the wormhole is at $\partial\Omega$. Because $\mathcal{M}$ is piecewise Minkowski space, the Reimann curvature is everywhere zero, except at the throat itself. At $\partial\Omega$ one expects the Reimann curvature tensor to be proportional to a delta function. This situation may be analysed by utilizing the "junction condition" formalism (also known as the "boundary layer" formalism) [6,7]. The boundary later formalism allows the Ricci tensor at the junction to be easily calculated in terms of the second fundamental form of $\partial\Omega$. The second fundamental form is

$$K_{ij} = \tfrac{1}{2}\, g^{ik} \cdot \left.\frac{\partial g_{kj}}{\partial \eta}\right|_{\eta=0}. \tag{2.1}$$

where $\eta$ denotes the normal coordinate to $\partial\Omega$. Then the Ricci tensor is

$$R^{\mu}{}_{\nu}(x) = -2 \begin{bmatrix} K^{i}{}_{j}(x) & 0 \\ 0 & K(x) \end{bmatrix} \cdot \delta(\eta). \tag{2.2}$$

The Einstein Hilbert action reduces to

$$S = \frac{c^3}{16\pi G} \int_{\mathcal{M}} \sqrt{g}\, R = \frac{\hbar}{16\pi L_P^2} \int_{\mathcal{M}} \sqrt{g}\, R = \frac{\hbar}{4\pi L_P^2} \int_{\partial\Omega} \sqrt{g}\, K. \tag{2.3}$$

where $L_P$ is the Planck length. The action has been reduced to an integral over the three geometry of the wormhole throat $\partial\Omega$. If we further specialise to the case of spherical symmetry the geometry is uniquely specified by a single degree of freedom: the radius of the throat $a(\tau)$. The second fundamental form is

$$K^i{}_j = \begin{bmatrix} K^\tau{}_\tau & 0 & 0 \\ 0 & K^\theta{}_\theta & 0 \\ 0 & 0 & K^\theta{}_\theta \end{bmatrix} \tag{2.4}$$

This has now reduced the computation of the Einstein Hilbert action to that of computing the two non trivial components of the second fundamental form. This calculation may conveniently be found in references 4 and 6 with the result that

$$K^\theta{}_\theta = K^\varphi{}_\varphi = \frac{1}{a}\cdot\sqrt{1+\dot{a}^2}; \qquad K^\tau{}_\tau = \frac{\ddot{a}}{\sqrt{1+\dot{a}^2}} = \frac{d\,\mathrm{Sinh}^{-1}(\dot{a})}{d\tau}. \tag{2.5}$$

Now noting that $\sqrt{g_3}\, d^3x \to 4\pi\, a^2\, d\tau$, we see that the Einstein Hilbert action is

$$S = \frac{\hbar}{L_P^2}\int\left\{2a\sqrt{1+\dot{a}^2} + a^2 \tfrac{d}{d\tau}\mathrm{Sinh}^{-1}(\dot{a})\right\} d\tau \tag{2.6}$$

After an integration by parts the gravitational Lagrangian restricted to this minisuperspace may be identified as

$$L = \frac{2\hbar}{L_P^2}\cdot\left\{a\,\dot{a}\,\mathrm{Sinh}^{-1}(\dot{a}) - a\sqrt{1+\dot{a}^2}\right\}. \tag{2.7}$$

It should be noted that the procedure described above is a direct analogue of the minisuperspace techniques more commonly used in quantum cosmology [8]. The radius of the wormhole is a direct analogue of of the scale factor of the universe, the different physics in this case arising from the different dependence of the Ricci scalar on $a(\tau)$.

## 3. Quantization.

Now that the Lagrangian is known, quantization is straightforward. The only remarkable aspect of the analysis is that closed form exact expressions are obtained. The conjugate momentum to $\dot{a}$ is

$$p \equiv \frac{\partial L}{\partial \dot{a}} = \frac{2ah}{L_P^2}\,\mathrm{Sinh}^{-1}(\dot{a}). \tag{3.1}$$

This relation may be inverted to yield $\dot{a} = \mathrm{Sinh}(\frac{L_P^2}{2ah}\cdot p)$, so that the Hamiltonian is

$$H(p,a) \equiv p\dot{a} - L = \frac{2h}{L_P^2}\cdot a\cdot \mathrm{Cosh}\left(\frac{L_P^2}{2ah}\cdot p\right). \tag{3.2}$$

While this is still a *classical* Hamiltonian which would seem to have tremendous factor ordering ambiguity, the factor ordering ambiguity may be removed in a natural way by demanding Hermiticity (self adjointness) of the Hamiltonian. In fact if we now replace $p \mapsto -ih\frac{\partial}{\partial a}$, the resulting Hamiltonian *operator* is seen to be

$$H = \frac{2h}{L_P^2}\cdot a\cdot \mathrm{Cos}\left(\frac{L_P^2}{2}\cdot\frac{1}{a}\frac{\partial}{\partial a}\right) \tag{3.3}$$

The wave function of the wormhole is determined in the usual fashion by the Wheeler de Witt equation $\hat{H}\psi(a) = 0$. (The fact that the "eigenvalue" is zero is a standard consequence of the reparameterization invariance of the theory). The solutions to this equation are easily seen to be

$$\psi_n(a) \propto \exp\left[-\tfrac{1}{2}(2n+1)\pi(a/L_P)^2\right]. \tag{3.4}$$

Here $n$ is an integer valued quantum number describing the internal state of the wormhole. Negative values of $n$ are not normalisable and so are discarded. (The appropriate normalization is $\int|\psi(a)|^2\,da = 1$). The expectation value of the wormhole radius is seen to be

$$\langle a \rangle \sim L_P / \sqrt{(2n+1)}. \tag{3.5}$$

Thus the wormhole has been quantum mechanically stabilized with a radius of order the Planck length.

A few words of discussion are in order:

Considerable insight into the meaning of these wavefunctions may be obtained by considering an analogy with the s-wave Hydrogen atom. A Hydrogen atom with zero angular momentum is classically unstable against collapse, but is quantum mechanically stable with a radius of order the Bohr radius. The wormhole system under consideration may be viewed as the gravitational analogue of this simple system. In particular the $n < 0$ modes that were trown away are directly analogous to the non normalizable modes of the quantum Hydrogen atom.

Perhaps the most potentially damaging criticism that can be made concerning this calculation is that it is performed in minisuperspace instead of using Wheeler's full superspace. It is quite possible that the brutal truncation from an infinite number of degrees of freedom $g_{ij}(\vec{x}, t)$ down to *one* degree fo freedom $a(\tau)$ has also brutally truncated the real physics. There is no entirely satisfactory answer to this objection, except to point out that given our current calculational abilities (or rather, lack of them) we simply have no choice if we are to be able to calculate anything. I would however remind the reader that although the application is unique, the minisuperspace technology employed is a standard quantum gravity technique.

The fact that the Wheeler de Witt equation for the wormhole is not quadratic in momenta is an unusual feature that may cause some confusion. This non quadratic behavior is however merely a reflection of our choice of canonical position variable $a(\tau)$

## 4. Conclusions.

In this note we have seen how quantum mechanics stabilizes Minkowski signature wormholes and prevents them from "pinching off". If this result is as general as expected it has implications for topology changing processes in quantum gravity. It suggests that the causality violations engendered in spacetime by classical topology changes may be prevented at a quantum mechanical level by this quantum stabilization process.

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